SERIES OF PROBLEMS, AT THE CROSSROAD OF RESEARCH, PEDAGOGY AND TEACHER TRAINING

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Our presentation is focused on the design of a favourable environment for teachers willing to develop disciplinary or interdisciplinary reflections based on the study of a particular kind of historical sources, that we call "series of problems". In the first part we briefly explain the meaning of this notion and we outline the contents and purpose of a research project focused on their comparative study. We then describe how this project is associated with a professional teacher training session also conceived as a research seminar. In the third part we explain how we intend to reorganise the project into a collaborative edition of a sourcebook about series of problems. We finally explain the main principles of a workshop associated with this editorial enterprise that will permit the development of original reflections and pedagogical projects around the texts that will be selected for the sourcebook.

"SERIES OF PROBLEMS AT THE CROSSROAD OF CULTURES": THE FIRST STAGE OF A RESEARCH PROJECT (2011-14)

The research project entitled “Series of problems at the crossroad of cultures” has been developed within the "HASTEC labex", a cluster of several Parisian research laboratories [1]. It gathers around 15 researchers, including master and PhD students, from various disciplines: history, epistemology and anthropology of sciences and of literature, history of texts, cultural history, and educational studies. The purpose of this interdisciplinary project is to study a genre of historical texts called "series of problems". Many (though not all) of these texts can be identified as having, partially or in totality, mathematical contents.

We use the term "series of problems" to interpret historical texts having the form of a collection of questions and answers. This interpretation relies on the basic hypothesis that these texts follow, either globally or locally, some kind of principle of ordering. The term "problem" has to be understood here in a very broad sense, as referring to any kind of verbal challenge: this includes, therefore, mathematical or scientific problems in the usual sense, but also riddles (enigmata) or questions, in general any kind of practical, pedagogical or intellectual "task". As for the term "answers", it also refers to a wide range of possibilities, from a "solution" (in the case of mathematical problems) to quotation of authorities (in the case of questions in natural philosophy) or poems (in the case of literary riddles). The main originality of this research project, then, consists in focusing not on individual ‘problems’ but
on the principles, the characteristics and the possible roles of their collection in a certain order. The second keypoint is that this order or "principle of ordering" is not necessarily understood in the same way in each case. In other words, the confrontation and progressive clarification of the various ways, in which this ordering should be understood, is one of the basic goals in this interdisciplinarily project.

A famous example of sources entering the generic category of "collection of questions and answers" and dating back to antiquity are the (pseudo) Aristotelian problemata, a series of questions falsely attributed to Aristotle and proposing a wide range of intellectual and philosophical challenges [2]. Another example, which is the research subject of one of us, is a corpus related to a Hungarian tradition of mathematics education from the second half of the 20th century: mainly textbooks and teachers handbooks, partly inspired by philosophical texts and by books popularising mathematics, like Rózsa Péter's "Playing with infinity". In these texts, ordered series of problems play an essential role, as well as a dialogical form of presentation. In this case, diverse principles of ordering can be observed, for example the variation of mathematically similar problems in different contexts, in order to guide the reader towards progressive levels of generalisation (Goszt Tonyi 2015).

Many other examples could be mentioned of course. Synoptic figure 1 gives an idea of the variety of texts and periods covered by the researchers participating in the project [3], and still many more could be added when considering other periods of cultural areas.

These texts are highly interesting research objects in several respects. Let us first insist on their interest for historical and anthropological studies. Some of these series, in their form and contents, have a long ranging history: this is, for example, the case of the pseudo-Aristotelian problemata, the tradition of which extends to the Middle Ages, during which they were eventually adapted, through reordering and adaptation, into a form of encyclopaedic knowledge (Ventura 2008). Some of them have crossed the boundaries of cultures, like Diophantus' series of arithmetical
problems: originated in Greek in a coherent treatise, they have been transmitted to Byzantine and Arabic Middle Ages, before they were translated and adapted in the Renaissance periods and inspired a host of new treatises—often by way of reordering and transforming Diophantus's problems. Thus the study of the re-appropriation and re-ordering of such texts contributes to the understanding of their long-lasting character.

At the same time, even in the case when they are based on a long-lasting heritage, they are never organised the same way and represent an intriguing object for cultural studies: thus, Rózsa Péter’s literary text can be seen as an initiation to mathematics for a wide audience, but also as a reflection of the philosophical, educational and literary concerns of a whole milieu of writers, mathematicians, educators and philosophers to whom R. Péter alludes in her book (Gosztonyi 2015). In this case, part of the book is based on problems structured in a sophisticated way, this ordering having much to do with the concerns in question. The pseudo-Aristotelian problemata, which cannot positively be attributed to Aristotle himself, still reflect the spirit and atmosphere of the peripatetic school. As for Diophantus' Arithmetica, it contains strong allusions to the background of ancient rhetoric, most notably the emphasis on the notion of invention. The latter is in turn related to the progressivity of his problems, which is meant to develop the reader's capacity for invention (Bernard 2011, Bernard and Christianidis, 2012). In general, these texts often pose difficult questions of interpretation: even when the intention behind their constitution is made explicit, it is not always obvious how to make it correspond to the actual structure of the text. Thus, looking into their partial or global "seriality" is one way (among others) to construct this interpretation and face this difficulty.

The third interest of such objects is the historiographical issues raised by their classification. For example, several of these series have been categorised by historians in a way that is open to dispute—in particular, while there are sometimes clues to the fact that they served didactic or pedagogical roles, in other case the positive evidence for this is lacking or, when it exists, is easily misinterpreted [4].

Finally, series of problems often represent a challenge for historical research, because in some cases (esp. in the medieval period), historical inventories of them are lacking, and many sources that fall under this category are still unedited or understudied. When an inventory is possible and expected, the criteria for building these inventories and comparing the elements of the retained corpus are also open to discussions: should the text be characterized through the contents; through the list of the statements of problems; or through the list of solutions? Finally, even in the case of the study of single series of problems, the criteria that make clear the organisation and ordering of the problems or questions have to be made clear and studied carefully, because this analytical choice has deep consequences on the interpretation of "seriality".
All in all, the primitive aim of the project has been, and still is, to improve the comparative study of these objects, not only across different ages and geographical locations, but also by taking advantage of the variety of approaches and fields represented in the research group. This variety of approaches focused on the one same object is one way to understand the notion of "crossroad of cultures": as the crossroad of intellectual approaches. Another way to understand this notion is to think about it in terms of the variety of "cultures", including intellectual and professional cultures and techniques, beyond differences of language and values, that are needed to understand "series of problems". We want to check the fundamental hypothesis that, given their complexity, taking into account several of these cultures and not only one or two of them, might bring a better historical understanding of their structure, role and relative stability in time.

Beyond their interest for historical studies, the second reason to pay attention to these objects is the fact that their study can still inspire new reflection by teachers of today working in various disciplines, or in interdisciplinarity. This is why the development of the project was very soon associated with a training session, as we shall now see.

THE ORGANIZATION AND CONCEPTION OF TRAINING SESSIONS AROUND SERIES OF PROBLEMS.

Series of problems can challenge the interest of teachers for several reasons. Some of them are related to the general issues usually treated in HPM meetings: as examples of historical sources among others, they are liable to inspire pedagogical activities and reflections on mathematics in relation to the cultural context or more specifically to the interest of reading mathematics into ancient and unfamiliar texts. Also, they are potentially interesting for interdisciplinary activities: on the level of contents, the problems contained in these texts do not all concern mathematics; on the level of their interpretation, series of problems are akin to a genre of texts, that is to an interpretative tool used in literary studies.

More specifically, though, teachers may find reasons for taking interest in this literature, that are related to the pedagogical issue of teaching through problems. By studying series of problems and reflecting on questions about the order of problems, one can take into account not only the resolution of isolated problems but also structured systems of problems as well as the intellectual processes consisting in putting them in order. At an even deeper level, there is also an issue about the image and conception of knowledge which is reflected through its organisation in this serial structure.

To discuss concretely such questions, we have organized three training sessions until now (2012-15). They have been proposed to a mixed audience of teachers of mathematics, literature and history in French secondary schools, as well as to students in the human sciences through HASTEC and the associated master or
doctoral structures [5]. The double purpose is (a) to offer an interesting incentive for professional development, for teachers willing to enrich their culture and knowledge, and their reflection on the teaching through problems or enigmas; and (b) to propose them simultaneously an insight into research questions about the historical sources taken as a support and point of departure of such reflections [6].

The session had each time a standard format: three days of meeting amounting to six sessions of 3 hours each. The first session is meant to expose the purpose and contents of the training, with a special emphasis on the ambivalence of the sessions, which can be seen as stages in an ongoing research seminar, and as opportunities to discuss professional issues. Then each of the other five sessions is based on the study of a particular corpus of series of problems, and consists in the presentation of their historical context on the one hand, and the collective reading of a collection of selected excerpts on the other. In principle, enough time must also be left each time for discussions with the participants on the contents of the proposed text and of its interpretation, but also on professional issues aroused by this experience of reading and understanding of ancient sources.

Let us insist here on the organisational aspects of theses sessions that make them a kind of concrete crossroad between professional and research inquiries. The first way to favour this mixture of perspectives has been already mentioned: the researchers (including students) who were called to constitute and present a collection of selected texts for discussion with the participants, were also invited to organise these excerpts according to one leading research question they had in mind. This presentation is basically meant to give an idea of the underlying research issues. For example, one of us took the opportunity of the 2013-14 session to explain the questions he had in mind about the progressivity of Diophantus's problems in his *Arithmetica*: having elaborated a first model of study of this progressivity (Bernard and Christianidis 2012) his purpose is now to improve this interpretative model through closer attention to the language used by Diophantus for the statements of his problems and the corresponding solutions (Bernard forthcoming).

Even more concretely, it very soon appeared that one way to explain the research questions was not only to explain it through a traditional kind of talk giving elements of theoretical references or historical context, but also to make it palpable through the organisation of the chosen texts. Following on the example of Diophantus's text, we were for example led to propose a translation of its problems so as to get the reading experience as close as possible to what it was in antiquity: an experience of mentalizing texts that were written in manuscripts in a "continuous" way (with no or little separation between words and sentences) through aloud reading and verbalization (*anagnōsis*). This way of preparing and presenting the text was meant to make clear and palpable the repetitiveness which is characteristic of ancient texts. It can then be explained by taking into account the concrete conditions in which reading and learning occurred in antiquity. Interestingly enough, this issue...
met very quickly the concerns of mathematics teachers who are developing ways to help the reading of sentences, in which algebraic symbols appear that are liable to be replaced by numerical values [7].

The other way, through which the participants were invited to participate in research inquiries and, at the same time, to question their own professional practice of concerns, is more traditional: it consists in explaining, from the outset, the origins and reasons of the session - especially the fact that it is related to a research project. At this stage the purpose is to organise a first discussion, aiming at "matching" the expectations of participants, with the purpose of the session. From this discussion generally emerges several questions and issues that participants have in mind consciously or not. Here are some examples of questions that typically emerged from such preliminary discussions: how to structure one's teaching through problems and for what purpose? How can one introduce a cultural context when discussing traditional problems? What use can be made of problems stemming from, or present in, various cultural traditions and cultures, especially in view of teaching mathematics in multicultural classes? How the same problems were formulated in various periods and languages, and what advantage can be drawn from this variety? The game, then, is to recall as systematically as possible these issues in the course of the various reading sessions. From this point of view, these sessions can be then regarded as a permanent anamnesis (recollection) of these key issues. This concretely calls for the presence of a moderator who should see his role as essentially maieutic, that is, as 'recollecting' the previously discussed questions. This means that to make bridges between them, reformulating them in the light of new contents, adding content, awakening new reactions and discussions, is for them the main challenge.

The limits of these procedures are, of course, time. While there is of course no limit of time for preparing a set of 'interesting' excerpts to study and read, there are obvious constraints on the time that can be devoted to open discussions in the framework of reading sessions, most of which are spent on discovering the presented material and on going beyond the usual first "shock" of meeting new and unfamiliar texts. Combining the presentation of the cultural context, open discussions and the reading of texts is a real difficulty. We thus naturally came to the idea that we should associate with these sessions a more intensive workshop. This idea also came from the recent transformation of the underlying research project, which is now turning into a reading seminar associated to an editorial project.

THE PROJECT OF A COLLABORATIVE "SOURCEBOOK" ON SERIES OF PROBLEMS

Before we come to explain how we are conceiving this associated workshop, the first edition of which began in 2014-15, we must explain the new direction the "series of
problems” research project is taking, toward a reading seminar aimed at the publication of a sourcebook for the subject.

After a first 'seminal' period of three years that is now concluding with a first collective publication (to appear in 2015), the "series of problems" project is progressively taking a new turn. Since most of the participants are now willing to have a reading seminar in which excerpts of various series of problems would be examined and discussed "from close experience", the basic idea is that this seminar would be ultimately focused on the publication of a sourcebook consisting of a collection of discussions and annotations of these excerpts.

In its traditional form for history of science, a sourcebook essentially proposes historical and epistemological commentaries on the chosen texts. In our project, this would constitute the first layer of the expected commentaries. But, based on a previous experience of a similar editorial project associated with an experience of collective reading of the chosen material by teachers (Bernard et alii, 2010), the originality of this sourcebook would be to add a second layer of commentaries. This second layer would account for the lessons drawn from concrete experience of "actual encounters" between these texts, and with teachers concerns with specific professional issues. The key idea underlying these second commentaries is reflected in the beautiful narrative that Augustine proposes of this conversion, in a well-known episode of his Confessions (VIII, 29): some texts achieve an actual meaning for their readers, through the identification of its contents with the actual experiences and thoughts of the reader. These commentaries, then, would illustrate possible values actually given to these ancient texts by modern readers, especially teachers.

When considered on the level of research questions, the two layers of commentaries correspond to two basic kinds of issues and purposes. The first layer is oriented on historical and epistemological research on the texts themselves: the leading purpose is then to restore the adequate historical and cultural context in which the chosen excerpt might or should be understood, the reasons for choosing the excerpt and to provide elements of interpretation based on actual research, which includes bibliographical references giving an access to deeper readings. The leading questions, then, are those exposed in the first part of the present paper. The second layer is explicitly or not, related to the issues in educational research about learning and teaching through problems that have been evoked in the previous part. From this second point of view, the key issues are the meaning of "teaching through problems"; the role of seriality when building problems is considered not as an isolated activity but as building collections of them with a definite idea in mind, whatever it is.

Concretely, the first layer of commentaries is naturally obtained through the existence of the reading seminar called for by the participants of the project, as mentioned above. The second layer requires a slightly different kind of context and framework: for this, the workshop to which we alluded above, in relation to the training session on series of problems, seems an adequate answer.
ORGANIZING WORKSHOPS FOR PROFESSIONAL DEVELOPMENT INSPIRED BY HISTORICAL "SERIES OF PROBLEMS".

The basic aim of the workshop in question, then, is to serve as a "companion" both to the training sessions and the project of the sourcebook we discussed above [8]. As for the participants, the idea basically follows the principle of the IREM workshops [9]: researchers and teachers at various levels (primary, secondary or university) are invited to participate, provided they have some connection with the associated research project. This open framework is wholly consistent with the purpose of mixing various research and professional perspectives in one and the same framework. For teachers, it should offer the right context for professional development, that is, an opportunity to reflect about their own teaching on the basis of the historical material discussed within the project.

As compared to traditional IREM groups in France, our workshop has the particularity to welcome teachers of literature and history, who might be interested by in this material. Moreover, it is in principle open to students of the newly introduced Master’s curriculum for all professions related to teaching and education (MEEF). In other words, it could become a place for meetings not only between researchers and teachers, but also between beginning and qualified teachers, and between teachers from different disciplines.

Just as we did above for the training session, we would like here to highlight how we conceived the organisation of the workshop, so as to fulfil its basic objectives. The same problems of conception mentioned above in relation to the training session, exist with this workshop in terms of organising a coherent dialogue between research perspectives and questions related to the development of professional skills. At the present moment, it is too early to draw conclusions from this nascent experience; we will limit ourselves to discussing the initial framework we considered for it, and the possible perspectives.

Our first idea is both traditional for an IREM-type workshop and an original development to our conception of the training session. Any IREM group has for its basic purpose the production of resources for mathematical teaching. Just in the same way, we thus propose that the participants develop a project that might evolve in as a possible resource for other colleagues: this might be for example an article, an academic work (a Master’s thesis for example), a website, or a booklet on a definite subject, etc.

It is important to leave significant freedom in terms of the potential nature and contents of the resource they propose to build. As for the kind of project, it might evolve into personal reflections on “series of problems” as cultural, historical, literary objects, in a typically interdisciplinary perspective. This might also consist in reflections on the ways of constructing a teaching process based on problems, these reflections being inspired by the examples seen during the training session. This
might be the construction and experimenting with teaching scenarios based on series of problems.

In order to leave place for the development of this diversity, and to define the projects which are the better adapted to the interests, competences and possibilities of each participants, we asked them, at the initial stage, to express some rationale, emotion or desire, that they could develop later into a more definite project. In other words, the first issue is not to define a project but to explore its roots, that is, the reasons for building a project. This notion has much to do with the Augustinian idea of "encounter with a text" underlined above.

For example, two colleagues already signalled their interest in the material that one of us proposed during the training session, in the form of translated excerpts from the Hungarian mathematics textbooks from the 1970s. What attracted their attention to this material was the fact, that part of the textbook took the form of fictive dialogues between pupils sharing their experiences and questions related to concrete problem-situations. [10] The discussion showed, that the reasons for being attracted to this idea were related to their own attempts to structure their teaching through the use of actual dialogues. More than this we cannot say at the present stage: we do not know yet, what use they made of dialogues and in what sense they understand this use. This might ultimately appear very different from the intentions that underlie the Hungarian texts; what counts at this stage, is that they began to identify the reasons for studying from close examination of these manuals, in relation to this particular professional experience.

The second leading idea is to leave time and freedom for the progressive development of the project. Time is again an obvious constraint: the participants, most often than not, do not have enough time and availability for developing a complete project with compelling deadlines in a short span of time. They certainly need a challenge, but not deadlines that would be incompatible with their professional activity. One of the main reasons to leave open the type of work and the support chosen is to make sure that the complexity, length and support of the project does correspond to the time constraints of the participants, and most of all that its nature and contents fit the initial desire analysed in the first place.

Ultimately, our hope and purpose is to build the concrete basis for the elaboration of the "teaching" commentaries we alluded to above: if the work led within this framework develops in the right way, it should ultimately be possible to build a synthesis making for each text the best out of various reading experiences.

**CONCLUSIONS AND PERSPECTIVES.**

Our initial incentive for presenting this nascent work in the Copenhagen conference was to take the opportunity of an international conference in order to check whether this project could be developed on a more European level. One purely potential reason for thinking about this kind of development is that we belong to two different
countries, France and Hungary: thus, while the present project is developed in France and more precisely in Paris, one could imagine in the future some 'satellite' development in Hungary, or in general in other countries.

One strong objection to such development is the language: at present, most (though not all) of the texts developed within the project, or discussed in the training sessions, were presented in French and for a French-speaking audience [11]. Thus, as usual, bringing the project at a European level would mean overcoming language barriers. It also implies difficulties in terms of finding locally enough experts available for participating in interdisciplinary discussions, not only with other specialists, but also with teachers.

In spite of this, it remains interesting to reflect about the potentials for delocalization. Given the principle of the workshop described above, there is nothing to prevent several similar groups develop in various locales, even within France for example. Indeed, what really counts is the availability of the texts studied, translated and eventually edited within (and thanks to) the project; and the possibility of inviting participants in the projects to local meetings. The development of such 'satellites' thus need time, patience and reflection.

References:

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Gosztonyi Katalin (forthcoming in 2015), "Séries de problèmes dans une tradition d’enseignement des mathématiques en Hongrie au 20e siècle". In Les séries de
problèmes, un genre au carrefour des cultures. SHS web of conferences. EDP Sciences.


1 “labex” is a shortcut for "laboratoire d'excellence". The list can be found on the following webpage: http://www.hesam.eu/labexhastec/partenaires/(accessed 29 May, 2015).


3 The initials refers to the various researchers involved, namely: Al. B = Alain Bernard (UPEC), Au. B = Aurélien Berra (Paris Ouest Univ.), GC = Giovanna Cifoletti (EHESS), JC = Jean Christianidis (Univ. of Athens), KG = Katalin Gosztonyi (Univ. Szeged), SL = Stéphane Lamassé (Paris 1), CM = Caroline Macé (Leuven Catholic Univ.), NM = Anathanasia Megremi (Univ. of Athens), MM = Marc Moyon (Univ. of Limoges), JO = Jeffrey Oaks (Univ. of Indianapolis), JP = Jeanne Peiffer (CNRS-CAK), Iolanda Ventura (CNRS-IRHT), BV = Bernard Vitrac (CNRS-CAK). Other contributors are not mentioned in this synoptic schema.

4 For a more detailed discussion on these issues, see the collective volume that one of us co-edited with Christine Proust (Bernard and Proust 2014), which contains several illustrations taken from various contexts.

5 Some participants belonging to the two categories. In general, though, it appeared difficult to invite teachers of history of literature to these training sessions, a difficulty which is probably due to the presence of the word "problem" in the title of the session.

6 In terms of institutional background, this means that the session was hosted by one Institute for Research on the Teaching of Mathematics (IREM, in this case the Paris IREM based on Paris-Diderot University), co-organised by HASTEC and also the new Institute for teacher training at Créteil (ESPE de Créteil) to which one of us belongs.

7 As is well known, it is difficult for students to learn how to differentiate the cases when one has to consider symbols as 'empty places' that serve to highlight the reformulation of a problem of an algorithmic procedure, from the cases where these same symbols are to be identified as "having values". This issue has strong similarities with the issue of correctly interpreting Diophantus's use of the potentialities of the Greek language for building schematic solutions of his problems.

8 The two projects are obviously coherent with each other, since traditional sourcebooks usually are the formalised and published version of the material used in academic courses.

10 The series of problems, in this case, is thus the series of questions brought about in the various stages of this dialogue, as well as the problems these pupils either discuss or (re)formulate.

11 Note, nevertheless, that following a workshop held in Athens in October 2014, an English summary of the main issues discussed within the project is now available: http://problemata.hypotheses.org/373 (accessed 29 May, 2015).